18.06.01: 'Dimension'

Lecturer: Barwick

Wednesday 03 February 2016
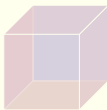
What *is* dimension?

Line segments are 1-dimensional; here's one now:

Planar regions are 2-dimensional; here's one:

Finally, cubes are 3-dimensional:

Question. What have these geometric figures to do with the numbers 1, 2, and 3? What are we counting when we count dimensions?

While you're thinking ...

Basic facts:

Lecturer: C. Barwick	
Website	http://math.mit.edu/~clarkbar/
Course website	https://stellar.mit.edu/S/course/18/sp16/18.06/
Email	clarkbar@gmail.com
Office location	building 2, office 476
Office hours	Tuesday and Thursday, 11 AM – NOON
Appointments	please book via website!

More administrative notes

A complete syllabus is available in paper form at the front of this room or in electronic form on the course website. Please read it carefully. Here are a few highlights:

(1) You must be registered for and attend a weekly recitation. This is a *critical* part of the course.

(2) The slides for each lecture will be posted on the course website. So if you have a communicable illness, please do *not* come to class, read the notes, and feel better soon!

(3) I'll put an approximate schedule for the class online. This is subject to slight changes.

(4) There will be weekly problem sets, four midterm exams, and a final exam.

(5) The problem sets will account for half of your grade. They are a primary mechanism for your learning in this class. Please hand them in on time! The lowest two homework grades will be dropped.

(6) Each of the five exams will account for a tenth of your grade. Your lowest midterm exam score will be replaced by your final score (if it is higher).

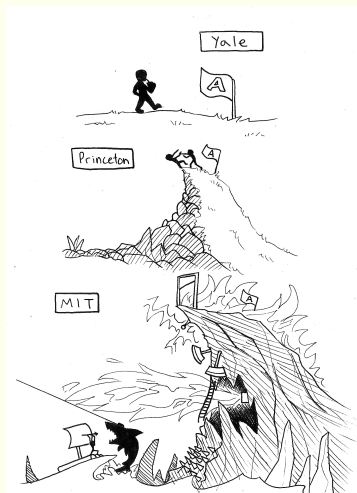(7) Do not be afraid to ask for help (with anything)! The sooner the better!

Figure 1: Grading policies at various schools. Not to scale.

Figure 2: Wants us to get back to the subject of dimension.

Dimension has to do with lines

1st approximation: "The dimension of a space is the maximum number of lines we can fit into the space."

This answer certainly works with lines: (Yup, looks good.)

It also works for points: points are 0-dimensional: ·

With planes, we need to be careful: I want to say that those two lines *define* the plane. That is, I want to say that if I take two lines, then there is *exactly one* plane that contains them, but that isn't quite right …

Two lines through a basepoint define a plane

If we have two lines that intersect at a *basepoint*: then there's a *unique*

plane that contains them. We call this the plane *spanned* by these lines.

The points of this plane are the corners of any parallelograms whose sides lie
on the given lines:

We say that any line in our plane passing through our basepoint is ***dependent*** upon our two starting lines: because it's completely contained in the plane spanned by these lines.

Dimension

This leads us to the following:

Definition. The dimension of a space is the is the maximum number of *independent* lines through a basepoint that we can fit into the space.

The word independent is important here. It means that no line lies in the space spanned by the other lines.

Definition. The dimension of a space is the minimum number of lines through a basepoint that *span* the entire space.

The most important fact you'll learn this semester is that *these are the same number*. Much of linear algebra comes from that!
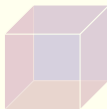
Now let's see how this plays out for our cube!

3-dimensional space
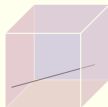
We draw a cube whose sides have finite length:

…but really I want you to think of it as infinitely big.
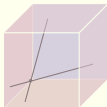
Now let's start sticking lines in there.

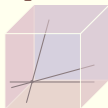(1) The first line we chuck in there however we like:
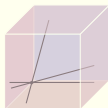
(2) The next line has to intersect the first:

(3) The third line to intersect the first two at their intersection point, but it is

not permitted to lie in the plane defined by the first two:

Now we have to stop! Any other line through our basepoint will go through the corner of some *parallelopiped* whose edges lie on our chosen three lines.

Any other line would thus be *dependent* on the three we've chosen. In other words, the three lines we've chosen *span* our infinite cube. In still other words, cubes are 3-dimensional.

So what?

We've found definitions that work for the flat figures we can draw:

Definition. The dimension of a space is the is the maximum number of *independent* lines through a basepoint that we can fit into the space.

Definition. The dimension of a space is the minimum number of lines through a basepoint that *span* the entire space.

What's amazing about linear algebra is that you don't have to be able to draw in order to make sense of this. It makes just as much sense to speak of 937-dimensional space as it does to speak of 3-dimensional space. The ideas are precisely the same.

What's even more incredible is that the numerics of dimension have the power to say really nontrivial things about systems of linear equations.

On Friday, what we'll do is to translate our geometric picture here into algebraic expressions. This will force us to introduce vectors. Please read §1.1 of Strang.