18.06 Exam III: Orthogonalize this!

6 April 2016

NAME: ______________________________

RECITATION: ____________________

GRADING	
1. _______	/20
2. _______	/20
3. _______	/20
4. _______	/20
5. _______	/20
TOTAL	/100

1. Veracious or fallacious

For each of the following sentences, indicate whether they are true or false. (No need to justify your answer.)

(a) If $\vec{v} \in \mathbf{R}^n$ is a vector and $W \subseteq \mathbf{R}^n$ is a vector subspace, then the projection $\pi_W(\vec{v}) = \vec{0}$ if and only if, for any vector $\vec{w} \in W$, one has $\vec{v} \cdot \vec{w} = 0$.

(b) If $\vec{v} \in \mathbf{R}^n$ is a vector and $W \subseteq \mathbf{R}^n$ is a vector subspace, then
$$\|\pi_W(\vec{v})\| \leq \|\vec{v}\|.$$

(c) Two vector subspaces $V, W \subset \mathbf{R}^n$ such that $V \cap W = \{\vec{0}\}$ are othrogonal.

(d) Any vector subspace $W \subseteq \mathbf{R}^n$ has an orthonormal basis.

(e) The only orthonormal basis of $\mathbf{R}^n$ is the standard basis $\hat{e}_1, \ldots, \hat{e}_n$.

2. Solve

Find an orthogonal basis for the space of solutions to the following system of linear equations in the five variables u, v, w, x, y:

$$\begin{aligned} u + w + y &= 0 \\ v + x &= 0 \end{aligned}$$

3. Is this projection accurate?

What is the projection of the vector $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \in \mathbf{R}^3$ onto the plane $3x - 4y + z = 0$?

4. More projecting

Compute the projection of the vector $\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \in \mathbf{R}^5$ onto the image of the following matrix:

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 1 \\ 1 & -1 & 0 \end{pmatrix}.$$

5. Householder

Suppose $\widehat{x} \in \mathbf{R}^n$ a unit vector. Write

$$N = \{\vec{v} \in \mathbf{R}^n \mid \vec{v} \cdot \widehat{x} = 0\} \subset \mathbf{R}^n.$$

This N is an $(n-1)$-dimensional vector subspace of $\mathbf{R}^n$. Also, write H for the $n \times n$ matrix $I - 2\widehat{x}\widehat{x}^\intercal$.

Prove that the projection $\pi_N(\vec{w})$ of $\vec{w}$ onto N is equal to the projection $\pi_N(H\vec{w})$ of $H\vec{w}$ onto N.