18.06 Spring 2012 – Problem Set 1

This problem set is due Thursday, February 16th, 2012 at 4pm (hand in to Room 2-106). The textbook problems are out of the 4th edition. For computational problems, please include a printout of the code with the problem set (for MATLAB in particular, `diary(‘‘filename’’)` will start a transcript session, `diary off` will end one.)

Every problem is worth 10 points.

1. Do Problem 8 from Section 1.3.
2. Do Problem 8 & Problem 32 from Section 2.2.
3. Do Problem 22 from Section 2.3.
4. Do Problem 19 & Problem 36 from Section 2.4.
5. For which values of q (if any) is the following system consistent (= solvable)?

$$x + 4y + 3z = 1,$$
$$q^3x + 4q^3y + 3q^3z = 64q.$$

6. A permutation matrix P comes from permuting the rows of the identity matrix I_n. If the entries of P are labelled p_{ij}, the matrix A having entries $a_{ij} = p_{ji}$ is the transpose, $A = P^T$.
   (a) Is P invertible, and if yes *why*? How would we proceed in Gaussian elimination on P?
   (b) Explain why the product $C = PP^T$ is the identity matrix. Think about where the 1's and 0's are.
   (c) Since the answer to (a) was "yes", what is the inverse to P?
7. (a) Give examples of non-zero (meaning: not all entries zero) 2×2 and 4×4 matrices A, one of each, such that $A^2 = O$ (recall O means the zero matrix). Hint: You only need to use one 1, and the rest of the entries can be 0's!
   (b) Are there any invertible $n \times n$ matrices A such that $A^2 = O$?
8. Given the three vectors $\mathbf{a}_1 = (1, 2, 3)$, $\mathbf{a}_2 = (1, 0, -1)$ and $\mathbf{a}_3 = (0, 0, 1)$, find (if possible) numbers x_1, x_2 and x_3 such that:

$$x_1\mathbf{a}_1 + x_2\mathbf{a}_2 + x_3\mathbf{a}_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$

   Your solution should involve Gaussian elimination on $A = \begin{bmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 \end{bmatrix}$ (the matrix with $\mathbf{a}_i$'s as columns).

9. (a) Using MATLAB, perform the matrix products A^2, A^3 and A^6 of the following lower-triangular matrix:

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 7 & 2 & 0 & 0 \\ 5 & 1 & 3 & 0 \\ 3 & 2 & -1 & 4 \end{bmatrix}$$

   (b) Explain the rule for *diagonal* entries of A^k, for a lower-triangular matrix A.

   (c) Guess a rule for the $(2,1)$ entry of A^k, for a lower-triangular matrix A.

10. A chemistry professor claimed on live TV that he could, by mixing, obtain *any* wine with given contents of water (W), sugar (S) and tannic acid (T), labelled by vectors $w = (W, S, T)$ such that $W + S + T = 100\%$. Due to a lack of research funding, his stock was quite limited:

    - Laboratory water supply: $w_1 = (100, 0, 0)$.
    - Budget wine: $w_2 = (50, 0, 50)$.
    - Plum tea concentrate: $w_3 = (30, 50, 20)$.

    (a) If a Chateaux Bordeaux 1915 has $(W, S, T) = (45, 50, 5)$, why was the professor *not* able to obtain this wine by mixing w_1, w_2, w_3? Explain by computing the mixing ratios needed (by MATLAB or by hand).

    (b) Help the professor restore honor, by adding any new wine w_4 that will enable him to make the Chateaux Bordeaux 1915 (a Chateaux Bordeaux 1915 not allowed!).

    (c) Are the mixing ratios unique after addition of the fourth wine?