18.06 Spring 2012 – Problem Set 10 (not handed in/not graded)

This short extra problem set is *not* to be handed in. The problems are meant to help you learn about linear transformations. The textbook problems are out of the 4th edition.

1. Do Problem 30 from Section 7.1.

   Solution. The radius vectors of the circle are $(0,1)$ and $(0,1)$ and these vectors go to the vectors $(\sqrt{3},1)$ and $\left(-\frac{1}{2},\frac{\sqrt{3}}{2}\right)$ □

2. Do Problem 31 from Section 7.1.

   Solution. A rectangular can be considered as a set

   $$R = \{(x_0,y_0) + c(x_1,y_1) + d(x_2,y_2) : 0 \leq c,d \leq 1\}$$

   where(x_0,y_0), (x_1,y_2) and (x_2,y_2) are the vertices of the rectangular and (x_0,y_0) is in between the other two vertices (x_1,y_1) and (x_2,y_2). The image $T(R)$ of R is

   $$T(R) = \{T(x_0,y_0) + cT(x_1,y_1) + dT(x_2,y_2) : 0 \leq c,d \leq 1\},$$

   which is a parallelogram or a line (with the assumption $T \neq 0$). The set $T(R)$ is a line iff T is not invertible.

   □

3. Do Problem 27 from Section 7.2.

   Solution. There is a linear transformation S such that $S(w_i) = v_i$ and S is the inverse of T. Hence T is invertible. □

4. Do Problem 32 from Section 7.2.

   Solution. False. If these n vectors are linearly dependant, there is a vector w which is not a linear combination of these n vectors and $T(w)$ is not determined by $T(v)$. □

18.06 Wisdom. Get as ready as you can for the finals, by doing as many old exams as you have time for (found on the 18.06 website under "Past Courses"). Strive to master the concepts and techniques of Linear Algebra - you will need it, both here and beyond.