18.06 Professor Strang Quiz 2 April 2, 2008

Your PRINTED name is: _____

Grading	
1	
2	
3	

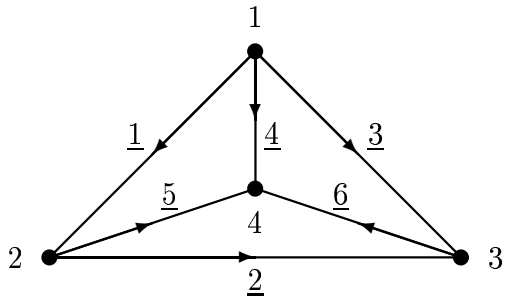
Please circle your recitation:

1)	M 2	2-131	A. Ritter	2-085	2-1192	`afr`
2)	M 2	4-149	A. Tievsky	2-492	3-4093	`tievsky`
3)	M 3	2-131	A. Ritter	2-085	2-1192	`afr`
4)	M 3	2-132	A. Tievsky	2-492	3-4093	`tievsky`
5)	T 11	2-132	J. Yin	2-333	3-7826	`jbyin`
6)	T 11	8-205	A. Pires	2-251	3-7566	`arita`
7)	T 12	2-132	J. Yin	2-333	3-7826	`jbyin`
8)	T 12	8-205	A. Pires	2-251	3-7566	`arita`
9)	T 12	26-142	P. Buchak	2-093	3-1198	`pmb`
10)	T 1	2-132	B. Lehmann	2-089	3-1195	`lehmann`
11)	T 1	26-142	P. Buchak	2-093	3-1198	`pmb`
12)	T 1	26-168	P. McNamara	2-314	4-1459	`petermc`
13)	T 2	2-132	B. Lehmann	2-089	2-1195	`lehmann`
14)	T 2	26-168	P. McNamara	2-314	4-1459	`petermc`

1 (33 pts.) (a) If $Ax = b$ and $A^{\mathrm{T}}y = 0$ then b is perpendicular to y. (The column space of A is perpendicular to the nullspace of A^{T}.) **Prove this by computing $(Ax)^{\mathrm{T}}y$.**

(b) Write down the 6 by 4 incidence matrix A of this graph (1 and -1 in each row of A). What is the dimension of the column space $C(A)$? Describe the nullpace $N(A)$.

(c) Find one nonzero vector $y = (y_1, y_2, \ldots, y_6)$ that is in the nullspace of A^{T}. (Think loops.) If voltages x_1, x_2, x_3, x_4 are assigned to the nodes (keep the x's as variables not numbers), multiply by A to find Ax. **Check that this Ax is perpendicular to your vector y.** (That's Kirchhoff's Voltage Law.)



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2 (33 pts.) (a) Suppose you want to fit the best straight line $C + Dt$ to the values $b = 1, 1, 1, 2$ at the times $t = 0, 1, 3, 4$. What is the matrix A in the unsolvable system $A \begin{bmatrix} C \\ D \end{bmatrix} = b$? Find the best $\widehat{C}, \widehat{D}$ and the heights p_1, p_2, p_3, p_4 of that line $\widehat{C} + \widehat{D}t$ at the times $t = 0, 1, 3, 4$.

(b) Think of the same problem as a projection onto the column space of A in $\mathbf{R}^4$. What is the error vector $e = b - p$? Show with numbers that e is perpendicular to (what space?).

(c) Use Gram-Schmidt to get orthonormal columns q_1, q_2 from the columns a_1, a_2 of your matrix A.

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3 (34 pts.) This question is about the matrix

$$A = \frac{1}{2}\begin{bmatrix} -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \end{bmatrix}.$$

(a) Compute A^2 and use that to show that the determinant of A is either 1 or -1.

(b) Determine whether $\det A = 1$ or -1.

(c) Find the cofactor C_{11} corresponding to the entry $a_{11} = -\frac{1}{2}$.

(d) Out of the $4! = 24$ terms in the "big formula" for $\det A$, show **four terms** that are $+\frac{1}{16}$. (For each term give the column numbers like $4, 3, 2, 1$ or $2, 1, 4, 3$ as you go down the matrix.)

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