18.06 Professor Strang Quiz 1 March 3, 2008

Grading

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Your PRINTED name is: ____________________

Please circle your recitation:

1)	M 2	2-131	A. Ritter	2-085	2-1192	`afr`
2)	M 2	4-149	A. Tievsky	2-492	3-4093	`tievsky`
3)	M 3	2-131	A. Ritter	2-085	2-1192	`afr`
4)	M 3	2-132	A. Tievsky	2-492	3-4093	`tievsky`
5)	T 11	2-132	J. Yin	2-333	3-7826	`jbyin`
6)	T 11	8-205	A. Pires	2-251	3-7566	`arita`
7)	T 12	2-132	J. Yin	2-333	3-7826	`jbyin`
8)	T 12	8-205	A. Pires	2-251	3-7566	`arita`
9)	T 12	26-142	P. Buchak	2-093	3-1198	`pmb`
10)	T 1	2-132	B. Lehmann	2-089	3-1195	`lehmann`
11)	T 1	26-142	P. Buchak	2-093	3-1198	`pmb`
12)	T 1	26-168	P. McNamara	2-314	4-1459	`petermc`
13)	T 2	2-132	B. Lehmann	2-089	2-1195	`lehmann`
14)	T 2	26-168	P. McNamara	2-314	4-1459	`petermc`

1 (18 pts.) Start with an invertible 3 by 3 matrix A. Construct B by subtracting 4 times row 1 of A from row 3. **How do you find B^{-1} from A^{-1}?** You can answer in matrix notation, but *you must also answer in words*—what happens to the columns and rows?

2 (24 pts.) Elimination on A leads to U:

$$Ax = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 3 & 3 \\ 1 & 3 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad \text{leads to} \quad Ux = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

(a) Factor the first matrix A into $A = LU$ and also into $A = LDL^{\mathrm{T}}$.

(b) Find the inverse of A by Gauss-Jordan elimination on $AA^{-1} = I$ or by inverting L and D and L^{T}.

(c) If D is diagonal, show that LDL^{T} is a symmetric matrix for every matrix L (square or rectangular).

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3 (30 pts.) Suppose the nonzero vectors a_1, a_2, a_3 point in different directions in R^3 but

$$3a_1 + 2a_2 + a_3 = \text{zero vector}\,.$$

The matrix A has those vectors a_1, a_2, a_3 in its columns.

(a) Describe the nullspace of A (all x with $Ax = 0$).

(b) Which are the pivot columns of A?

(c) I want to show that *all* 3 by 3 matrices with

$$(*) \qquad 3(\text{column } 1) + 2(\text{column } 2) + (\text{column } 3) = \text{zero vector}$$

form a **subspace** S of the space M of 3 by 3 matrices. Now the zero matrix is certainly included.

Suppose B and C are matrices whose columns have this property **(∗)**. To show that we have a subspace, we have to prove that every linear combination of B and C ___(finish sentence)___ .

Go ahead and prove that.

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4 (28 pts.) Start with this 2 by 4 matrix:

$$A = \begin{bmatrix} 2 & 3 & 1 & -1 \\ 6 & 9 & 3 & -2 \end{bmatrix}$$

(a) Find all special solutions to $Ax = 0$ and **describe the nullspace** of A.

(b) Find the complete solution—meaning all solutions (x_1, x_2, x_3, x_4)—to

$$Ax = \begin{bmatrix} 2x_1 + 3x_2 + x_3 - x_4 \\ 6x_1 + 9x_2 + 3x_3 - 2x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}.$$

(c) When an m by n matrix A has rank $r = m$, the system $Ax = b$ can be solved for which b (best answer)? How many special solutions to $Ax = 0$?

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