18.06 Problem Set 2

Due Wednesday, 20 February 2008 at 4 pm in 2-106.

Problem 1: a) Do problem 5 from section 2.4 (pg. 65) in the book.
b) Do problem 26 in section 2.4 (pg. 69).

Problem 2: Do problem 24 from section 2.4 (pg. 68).

Problem 3: Do problem 7 from section 2.5 (pg. 79).

Problem 4: Define the matrix

$$A = \begin{pmatrix} 1 & 2 & -4 \\ -1 & -1 & 5 \\ 2 & 7 & -3 \end{pmatrix}$$

Using elimination one can calculate that the inverse is

$$A^{-1} = \begin{pmatrix} -16 & -11 & 3 \\ \frac{7}{2} & \frac{5}{2} & -\frac{1}{2} \\ -\frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{pmatrix}$$

a) Suppose that we formed B by switching the top two rows of A. What would B^{-1} be?

b) Now suppose we defined C by adding three times column(3) of A to column(2). What is C^{-1}?

Problem 5: Do problem 23 from section 2.5 (pg. 80).

Problem 6: Do problem 29 from section 2.5 (pg. 81). As with any True/False question, be sure to explain your reasoning: give a brief proof if the statement is true, and give a counterexample if the statement is false.

Problem 7: Do problem 12 from section 2.6 (pg. 92).

Problem 8: Do problem 13 in section 2.6 (pg. 93).

Problem 9: Do problem 28 in section 2.6 (pg. 95).

Problem 10: In this problem we will use Matlab to do LU factorizations. Don't forget to include your code! Define the matrix

$$A = \begin{pmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{pmatrix}$$

The command **[L,U]=lu(A)** will decompose A into L and U. We can further decompose U by using the fact that A is symmetric, so that $U = DL'$ (the ' denotes transpose in Matlab). What are L, D, U? What will the pattern be for larger matrices of the same form?

Now, factor **B = [1,2;2,5]** into $B = C'C$ by using **C = chol(B)** (here **chol** stands for Cholesky). Try using the command **[L,U]=lu(B)**. What happens and why? We'll need to include a permutation matrix P via the command **[L,U,P]=lu(B)**. Find L, U, P and check that $PB = LU$.