18.06 Professor Strang Quiz 1 February 28, 2005

Grading

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Your PRINTED name is: ____________________

Please circle your recitation:

1)	M 2	2-131	A. Chan	2-588	3-4110	alicec
2)	M 3	2-131	A. Chan	2-588	3-4110	alicec
3)	M 3	2-132	D. Testa	2-586	3-4102	damiano
4)	T 10	2-132	C.I. Kim	2-273	3-4380	ikim
5)	T 11	2-132	C.I. Kim	2-273	3-4380	ikim
6)	T 12	2-132	W.L. Gan	2-101	3-3299	wlgan
7)	T 1	2-131	C.I. Kim	2-273	3-4380	ikim
8)	T 1	2-132	W.L. Gan	2-101	3-3299	wlgan
9)	T 2	2-132	W.L. Gan	2-101	3-3299	wlgan

1 (26 pts.) Suppose A is reduced by the usual row operations to

$$R = \begin{bmatrix} 1 & 4 & 0 & 2 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Find the complete solution (if a solution exists) to this system involving the original A:

$$Ax = \text{ sum of the columns of } A.$$

2 (18 pts.) Suppose the 4 by 4 matrices A and B have the *same column space.* They may not have the same columns!

(a) Are they sure to have the same number of pivots? YES NO WHY?

(b) Are they sure to have the same nullspace? YES NO WHY?

(c) If A is invertible, are you sure that B is invertible? YES NO WHY?

3 (40 pts.) (a) Reduce A to an upper triangular matrix U and carry out the same elimination steps on the right side b:

$$\begin{bmatrix} A & b \end{bmatrix} = \begin{bmatrix} 3 & 3 & 1 & b_1 \\ 3 & 5 & 1 & b_2 \\ -3 & 3 & 2 & b_3 \end{bmatrix} \longrightarrow \begin{bmatrix} U & c \end{bmatrix}$$

Factor the 3 by 3 matrix A into LU = (lower triangular)(upper triangular).

(b) If you change the last entry in A from 2 to ______ (what number gives $A_{\mathbf{new}}$?) then $A_{\mathbf{new}}$ becomes singular. *Describe its column space exactly.*

(c) In that singular case from part (b), what condition(s) on b_1, b_2, b_3 allow the system $A_{\mathbf{new}}x = b$ to be solved?

(d) *Write down the complete solution to* $A_{\mathbf{new}}x = \begin{bmatrix} 3 \\ 3 \\ -3 \end{bmatrix}$ (the first column).

4 (16 pts.) Suppose the columns of a 7 by 4 matrix A are linearly independent.

(a) After row operations reduce A to U or R, how many rows will be all zero (or is it impossible to tell)?

(b) What is the row space of A? Explain why this equation will surely be solvable:

$$A^{\mathrm{T}}y = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$