

## 18.06 - Spring 2005 - Problem Set 3

This problem set on lectures 7 – 9 is due Wednesday (February 23th), at 4 PM, at 2-106. Make sure to include your **name and recitation number** in your homework! The numbers of the sections and exercises refer to “Introduction to Linear Algebra, **3rd Edition**, by Gilbert Strang.”

Please staple your solution as first page of your homework. Remember to PRINT your name, Recitation number and Instructor name.

Lecture 7:

- **Read:** book section 3.2.
- **Work:** book section 3.2 (exercises 9, 15, 18, 20, 23, 27, and 28).

Lecture 8:

- **Read:** book section 3.3.
- **Work:** book section 3.3 (exercises 8, 13, 17, 18, and 19).

Lecture 9:

- **Read:** book section 3.4.
- **Work:** book section 3.4 (exercises 1, 6, 10, 24 and 31).

### Challenge Problem 1

Suppose  $R$  (an  $m \times n$  matrix) is in row reduced echelon form  $\begin{pmatrix} I & F \\ 0 & 0 \end{pmatrix}$ , with  $r$  nonzero rows and first  $r$  pivot columns.

- Describe the column space and nullspace of  $R$ .
- Do the same for the  $m \times 2n$  matrix  $B = \begin{pmatrix} R & R \end{pmatrix}$ .
- Do the same for the  $2m \times n$  matrix  $C = \begin{pmatrix} R \\ R \end{pmatrix}$ .
- Finally, do the same for the  $2m \times 2n$  matrix  $D = \begin{pmatrix} R & R \\ R & R \end{pmatrix}$ .

### Challenge Problem 2

- Suppose that  $A$  is a  $3 \times 3$  matrix. What relation is there between the nullspace of  $A$  and the nullspace of  $A^2$ ? How about the nullspace of  $A^3$ ?
- The set of polynomials of degree at most four in the variable  $x$  is a vector space. What is the nullspace of  $\frac{d^2}{dx^2}$ ? What is the nullspace of  $\left(\frac{d^2}{dx^2}\right)^2$ ?