18.06 - Spring 2005 - Problem Set 2

This problem set on lectures 4 – 6 is due Wednesday (February 16th), at 4 PM, at 2-106. Make sure to include your **name and recitation number** in your homework! The numbers of the sections and exercises refer to " Introduction to Linear Algebra, **3rd Edition**, by Gilbert Strang.".

Please staple your solution as first page of your homework. Remember to PRINT your name, Recitation number and Instructor name.

Lecture 4:

- **Read:** book sections 2.5 and 2.6.
- **Work:** book section 2.5 (exercises 8, 23, 30, 32 and 35) and 2.6 (6, 10, 13, and 20).

Lecture 5:

- **Read:** book section 2.7.
- **Work:** book section 2.7 (exercises 4, 12, 13, 17 and 40).

Lecture 6:

- **Read:** book section 3.1.
- **Work:** book section 3.1 (exercises 5, 10, 18, 23 and 24).

Challenge Problem

The inverse (add with E_{ij}^{-1} instead of subtract with E_{ij}) of an elementary elimination matrix is the identity with $+\ell_{ij}$ added in the i, j position. The magic of $A = LU$ is that multiplying those E_{ij}^{-1} in reverse order puts every ℓ_{ij} unchanged into L.

The problem is to prove this key Lemma:

> Suppose the matrix M has the ℓ's filled in up to but not including (i, j), and N is the next matrix with that next ℓ_{ij} filled in. Both have 1's down the main diagonal, and columns before j are all filled in. PROVE THAT $ME_{ij}^{-1} = N$.

Then every E_{ij}^{-1} fills in its ℓ_{ij} and the product of them all is the correct lower triangular L. Notice that E_{ij}^{-1} is multiplying *on the right*—what does that do to the *columns* of a matrix?