

18.06 Midterm Exam 2, Spring, 2001

Name _____

Optional Code _____

Recitation Instructor _____

Email Address _____

Recitation Time _____

This midterm is closed book and closed notes. No calculators, laptops, cell phones or other electronic devices may be used during the exam.

There are 3 problems. Good luck.

1. (40pts.) Consider the matrix

$$A = \begin{pmatrix} 1 & 0 & -1 \\ 3 & 1 & -1 \\ 9 & 5 & 1 \\ 9 & 8 & 7 \end{pmatrix}$$

- (a) Find the rank of A .
- (b) Find a basis for the row space of A , and find a basis for the nullspace of A . What is the dimension of the nullspace of A ?
- (c) What can you say about the relation between the rank and the dimension of the nullspace of A ?
- (d) Verify that all vectors in your basis of the nullspace are orthogonal to all vectors in your basis of the row space.

2. (30pts.) Let $a, b \in \mathbb{R}$, and let

$$A = \begin{pmatrix} 1 & 2 & 3 & a \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & b \end{pmatrix}.$$

- (a) What are the dimensions of the four subspaces associated with the matrix A ? This will of course depend on the values of a and b , and you should distinguish all different cases.
- (b) For $a = b = 1$, give a basis for the column space of A . Is this also a basis for \mathbb{R}^3 ? Justify your answer.

3. (30pts.) An experiment at the seven times $t = -3, -2, -1, 0, 1, 2, 3$ yields the consistent result $b = 0$, except at the last time ($t = 3$), when we get $b = 28$. We want the best straight line $b = C + Dt$ to fit these seven data points by least squares.

- (a) Write down the equation $A\mathbf{x} = \mathbf{b}$ with unknowns C and D that would be solved if a straight line exactly fit the data.

- (b) Use the method of least squares to find the best fit values for C and D .
- (c) This problem is really that of projecting the vector $\mathbf{b} = (0, 0, 0, 0, 0, 0, 28)^T$ onto a certain subspace. Give a basis for that subspace, and give the projection \mathbf{p} of \mathbf{b} onto that subspace.