MIT 18.06 Exam 1, Fall 2017
Johnson

Your name: ______________________________

Recitation: ______________________________

problem	**score**
1	/30
2	/20
3	/30
4	/20
total	/100

Problem 1 (30 points):

You are given three vectors $\vec{v}_1 = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$, $\vec{v}_2 = \begin{pmatrix} 1 \\ 0 \\ 5 \end{pmatrix}$, and $\vec{v}_3 = \begin{pmatrix} 0 \\ 2 \\ 4 \end{pmatrix}$. Your goal is to find a *linear combination of these three vectors* (that is, multiply them by some numbers x_1, x_2, x_3 and add them) to give the vector $\vec{b} = \begin{pmatrix} 2 \\ -2 \\ 12 \end{pmatrix}$.

(a) Write the equation in matrix form.

(b) Solve it to find the correct linear combination (x_1, x_2, x_3) of $\vec{v}_1$, $\vec{v}_2$, and $\vec{v}_3$.

(c) Change *one number* in $\vec{v}_3$ to make the problem have *no* solution for *most* vectors $\vec{b}$, but give a new vector $\vec{b}'$ for which there *is* still a solution. This new $\vec{b}'$ is in the ________________ space of the matrix________________.

(There are multiple correct answers for your new $\vec{v}_3$ and your new $\vec{b}'$.)

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Problem 2 (20 points):

Suppose A is some 3×3 matrix. We will transform this into a *new* 3×3 matrix B by doing operations on the rows or columns of A as follows. For each part, (i) **explain how to express B as B=AE or B=EA (say which!) for some matrix E (write down E!)**. Also, (ii) say **whether E is invertible** (that is, whether the transformation is reversible). (You don't need to compute E^{-1}, just say whether the inverse exists!)

(a) Swap the first and second rows of A.

(b) Keep the first row the same, *then* add the second row to the third row, *then* replace the second row with the sum of the first and third rows.

(c) Subtract the first *column* from the second and third columns.

(blank page for your work if you need it)

Problem 3 (30 points):

Suppose you have a 3×3 matrix A satisfying $A = B^{-1}UL$ where

$$B = \begin{pmatrix} 1 & 2 & 1 \\ 3 & -1 & 1 \\ -2 & 0 & -1 \end{pmatrix}, \quad U = \begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}, \quad L = \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 1 & -2 & 1 \end{pmatrix}.$$

(a) The *second* column c of the matrix A^{-1} satisfies $Ac = b$ for what right-hand side b?

(b) The *second* column c of the matrix A^{-1} also satisfies $ULc = d$ for what right-hand side d?

(c) Compute the second column c of the matrix A^{-1}. (**Important:** you don't *have* to compute the inverse of any matrix!)

(blank page for your work if you need it)

Problem 4 (20 points):

In class and homework, we showed that multiplying two arbitrary $m \times m$ matrices, doing Gaussian elimination, or inverting an $m \times m$ matrix requires $\sim m^3$ arithmetic operations (that is, roughly proportional to m^3 for large m). We found that adding matrices, multiplying an $m \times m$ matrix by a vector, or solving an $m \times m$ upper/lower triangular system of equations requires $\sim m^2$ operations.

Suppose that A is an $m \times m$ matrix, x is an m-component *column* vector (an $m \times 1$ matrix), and r is an m-component *row* vector (a $1 \times m$ matrix).

- You could compute the same result $xrAx$ by doing the multiplications in different orders, for example $x(r(Ax))$ (multiplying terms from *right to left*) or $((xr)A)x$ (multiplying from *left to right*). **Give the rough number of operations** (say whether proportional to $\sim m$, $\sim m^2$, $\sim m^3$, or $\sim m^4$) **for these two different orders (right to left and left to right).** Which one is the fastest for $m = 1000$?

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