18.06 Professor Edelman Quiz 1 October 5, 2011

Grading

1

Your PRINTED name is: ____________________

2

3

4

Please circle your recitation:

1	T 9	2-132	Kestutis Cesnavicius	2-089	2-1195	`kestutis`
2	T 10	2-132	Niels Moeller	2-588	3-4110	`moller`
3	T 10	2-146	Kestutis Cesnavicius	2-089	2-1195	`kestutis`
4	T 11	2-132	Niels Moeller	2-588	3-4110	`moller`
5	T 12	2-132	Yan Zhang	2-487	3-4083	`yanzhang`
6	T 1	2-132	Taedong Yun	2-342	3-7578	`tedyun`

1 (30 pts.)

Let $A = \begin{bmatrix} 0 & 0 \\ 6 & 9 \\ 2 & 3 \end{bmatrix}$.

(a) (6 pts.) Circle the best answer: The column space of A is a

a. point $\boxed{\text{b. line}}$ c. plane d. three dimensional space. Explain very briefly.

$\boxed{\text{The matrix has rank 1, seen by inspection or by elimination.}}$

(b) (6 pts.) True or $\boxed{\text{False}}$: The row space of A is a vector subspace of R^3, i.e., consists of a collection of vectors with three components that are closed under all linear combinations. Explain very briefly.

$\boxed{\text{The rowspace is a subspace of } R^2\text{, collections of vectors with } \textbf{two} \text{ components.}}$

(c) (6 pts.) Circle the best answer:

a. Matrix A has full column rank.

b. Matrix A has full row rank.

$\boxed{\text{c. Matrix } A \text{ has neither full column rank nor full row rank.}}$

Explain very briefly.

$\boxed{\text{The rank is both smaller than than the number of rows and the number of columns.}}$

(d) (12 pts.) Let $b = \begin{bmatrix} 0 \\ 3 \\ 1 \end{bmatrix}$. Find the complete solution to $Ax = b$.

The second column of A is free.

The particular solution is then $x_p = \begin{bmatrix} 1/2 \\ 0 \end{bmatrix}$. There is one special solution $\begin{bmatrix} -3/2 \\ 1 \end{bmatrix}$.

The complete solution consists of all vectors of the form

$$\begin{bmatrix} 1/2 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} -3/2 \\ 1 \end{bmatrix}.$$

This page intentionally blank.

2 (23 pts.)

A is a square 3×3 matrix whose LU decomposition exists with no row exchanges. Carefully provide a count of the exact number of operations required to compute the three parameters l_{12}, l_{13} and l_{23} of L and the six parameters of U. The questions below count first all the divisions, then all the multiplications, and then all the subtractions that occur.

Avoid any unnecessary operations. (Operations on the elements being eliminated are unnecessary since that element's ultimate fate is known to be 0.)

Note: Other answers maybe ok with explanation.

(a) (5 pts.) We recall that computation of each multiplier l_{ij} requires one division. The exact number of divisions in the LU decomposition of our 3×3 A is 3.

Each of the three multipliers requires one division to compute.

(b) (9 pts.) We recall that multipliers l_{ij} multiply the jth row but only to the right of column i. The exact number of multiplications in the entire LU decomposition of our 3×3 A is

$5 = 2^2 + 1^2$

The first row(column) is the pivot row(column) for the first two eliminations. First l_{21} scales A_{12} and A_{13} as a byproduct of the elimination of the (2,1) entry. Then l_{31} also scales A_{12} and A_{13}. By the time we have computed l_{32}, it multiplies the value now in A_{23}.

In general, we phrased the question in a way that counted for the fact that no computation need occur in the first row or first column. The total number of multiplications in general would then be $(n-1)^2 + \ldots + 1$.

(c) (9 pts.) We recall that after l_{ij} does its job of multipying row j to the right of column i, we subtract row j from row i but only to the right of column i. The exact number of subtractions in the entire LU decomposition of our 3×3 A is $\boxed{5}$.

$\boxed{5{=}2^2+1. \text{ The subtraction count is always the same as the multiplication count.}}$

3 (27 pts.)

A is a matrix which has two special solutions to $Ax = 0$. All other solutions, we recall, are linear combinations of the two special solutions. The two special solutions are

$$\begin{bmatrix} 3 \\ 1 \\ 4 \\ 0 \\ 5 \end{bmatrix} \text{ and } \begin{bmatrix} 2 \\ 0 \\ 2 \\ 1 \\ 2 \end{bmatrix}.$$

(a) (9 pts.) What is r =rank(A)? What is the dimension of the column space $C(A)$? What is the dimension of the nullspace $N(A)$? Very briefly explain your three numbers.

$r = 3 =$ (5 columns)-(2 special solutions)
dim($C(A)$)=$r = 3$
dim($N(A)$)=2=number of special solutions

(b) (9 pts.) B is a matrix that is the same as A except that its second row is (row 2 of A)-(row 1 of A). What is a basis for the nullspace $N(B)$?

The two special solutions above, since $N(B) = N(A)$.

(c) (9 pts.) C is a matrix that is the same as A except that its second column is (column 2 of A)-(column 1 of A). What is a basis for the nullspace $N(C)$? (Hint: If M is invertible, it may be useful to know that if y is in $N(C)$, then $M^{-1}y$ is in $N(CM)$.)

$C = AM$, where M is the matrix given in the hint. We recognize matrices such as M. The inverse of M looks the same as M except the $(2,1)$ entry is $+1$ not -1. M^{-1} takes a vector and replaces the first component with the sum of the first two components. Thus the new basis consists of the multiplication of M^{-1} times the previous basis vectors:

$$\begin{bmatrix} 4 \\ 1 \\ 4 \\ 0 \\ 5 \end{bmatrix} \text{ and } \begin{bmatrix} 2 \\ 0 \\ 2 \\ 1 \\ 2 \end{bmatrix}.$$

4 (20 pts.)

The next question concerns M_4, the 16 dimensional space of 4x4 real matrices.

(a) (10 pts.) True or $\boxed{\text{False}}$. The twenty-four 4 x 4 permutation matrices are independent members of M_4? Explain briefly.

At most 16 members of this space can be independent because it is a 16 dimensional vector

(b) (10 pts.) True or $\boxed{\text{False.}}$ The twenty-four 4x4 permutation matrices span M_4? (Hint: is any row sum possible?) Explain briefly.

To span the space, every matrix must be a linear combination of permulation matrices. This is impossible. Why? Because every row of a permutation matrix sums to 1, so every row of a linear combination of permutation matrices is the same. Try it! Thus a matrix with different row sums can not be a linear combination of permutation matrices.

This page intentionally blank.