18.06   Professor Edelman   Quiz 1   October 5, 2011

Your PRINTED name is: _____

Please circle your recitation:

1	T 9	2-132	Kestutis Cesnavicius	2-089	2-1195	kestutis
2	T 10	2-132	Niels Moeller	2-588	3-4110	moller
3	T 10	2-146	Kestutis Cesnavicius	2-089	2-1195	kestutis
4	T 11	2-132	Niels Moeller	2-588	3-4110	moller
5	T 12	2-132	Yan Zhang	2-487	3-4083	yanzhang
6	T 1	2-132	Taedong Yun	2-342	3-7578	tedyun

1 (30 pts.)

Let $A = \begin{bmatrix} 0 & 0 \\ 6 & 9 \\ 2 & 3 \end{bmatrix}$.

(a) (6 pts.) Circle the best answer: The column space of A is a

   a. point   b. line   c. plane   d. three dimensional space. Explain very briefly.

(b) (6 pts.) True or False: The row space of A is a vector subspace of R^3, i.e., consists of a collection of vectors with three components that are closed under all linear combinations. Explain very briefly.

(c) (6 pts.) Circle the best answer:

   a. Matrix A has full column rank.

   b. Matrix A has full row rank.

   c. Matrix A has neither full column rank nor full row rank.

   Explain very briefly.

2

(d) (12 pts.) Let $b = \begin{bmatrix} 0 \\ 3 \\ 1 \end{bmatrix}$. Find the complete solution to $Ax = b$.

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2 (23 pts.)

A is a square 3×3 matrix whose LU decomposition exists with no row exchanges. Carefully provide a count of the exact number of operations required to compute the three parameters l_{21}, l_{31} and l_{32} of L and the six parameters of U. The questions below count first all the divisions, then all the multiplications, and then all the subtractions that occur.

Avoid any unnecessary operations. (Operations on the elements being eliminated are unnecessary since that element's ultimate fate is known to be 0.)

(a) (5 pts.) We recall that computation of each multiplier l_{ij} requires one division. The exact number of divisions in the LU decomposition of our 3×3 A is _____.

(b) (9 pts.) We recall that multipliers l_{ij} multiply the jth row but only to the right of column i. The exact number of multiplications in the entire LU decomposition of our 3×3 A is

_____.

(c) (9 pts.) We recall that after l_{ij} does its job of multiplying row j to the right of column i, we subtract row j from row i but only to the right of column i. The exact number of subtractions in the entire LU decomposition of our 3×3 A is _____.

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3 (27 pts.)

A is a matrix which has two special solutions to $Ax = 0$. All other solutions, we recall, are linear combinations of the two special solutions. The two special solutions are

$$\begin{bmatrix} 3 \\ 1 \\ 4 \\ 0 \\ 5 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 2 \\ 0 \\ 2 \\ 1 \\ 2 \end{bmatrix}.$$

(a) (9 pts.) What is $r =\text{rank}(A)$? What is the dimension of the column space $C(A)$? What is the dimension of the nullspace $N(A)$? Very briefly explain your three numbers.

(b) (9 pts.) B is a matrix that is the same as A except that its second row is (row 2 of A)-(row 1 of A). What is a basis for the nullspace $N(B)$?

(c) (9 pts.) C is a matrix that is the same as A except that its second column is (column 2 of A)-(column 1 of A). What is a basis for the nullspace $N(C)$? (Hint: If M is invertible, it may be useful to know that if y is in $N(C)$, then $M^{-1}y$ is in $N(CM)$.)

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4 (20 pts.)

The next question concerns M_4, the 16 dimensional space of 4x4 real matrices.

(a) (10 pts.) True or False. The twenty-four 4 x 4 permutation matrices are independent members of M_4? Explain briefly.

(b) (10 pts.) True or False. The twenty-four 4x4 permutation matrices span M_4? (Hint: is any row sum possible?) Explain briefly.

Further hints for problem 3c.

Restatement of hint in a more suggestive notation: Assuming $C = AM$ and M invertible, y is in $N(A)$ exactly when $M^{-1}y$ is in $N(C)$.

The correct M to use in $C = AM$, is $M = \begin{bmatrix} 1 & -1 & & & \\ & 1 & & & \\ & & 1 & & \\ & & & 1 & \\ & & & & 1 \end{bmatrix}$. What is M^{-1}? What do you need to do with M^{-1}?