18.06 (Fall '11) Problem Set 6

This problem set is due Thursday, October 27, 2011 at 4pm. The problems are out of the 4th edition of the textbook. For computational problems, please include a printout of the code with the problem set (for MATLAB in particular, `diary(“filename”)` will start a transcript session, `diary off` will end one.)

1. Do problem 4 from 4.4.
2. Do problem 19 from 4.4.
3. Do problem 37 from 4.4. Hint: Find a vector in $c(A)$ that is orthogonal to $c(Q)$, then normalize.
4. Do problem 2 from 8.5.
5. Do problem 4 from 8.5.
6. Do problem 12 from 8.5.
7. (This problem is worth 20 points) In MATLAB or your favorite language, create $2n$-length discrete versions of $q_1 = 1/\sqrt{n}\cos(x)$ and $q_2 = 1/\sqrt{n}\cos(3x)$ by taking equal sized samples from $0 to 2\pi$, taking care to include 0 but exclude 2π. This means we want to think of each of these as column vectors $[x_0, \ldots, x_{2n-1}]^T$ where $x_i = i\pi/n$. In MATLAB this is `x=(0:(2*n-1))'*pi/n`. (before you go on, test to yourself that they're unit vectors). Let $Q = [q_1\ q_2]$.
   (a) Derive an identity for $\cos(3x)$ in terms of $\cos(x)$ (hint: you can use sum to product formulae). Use this identity to prove that $\cos(x)^3$ is in the span of $\cos(x)$ and $\cos(3x)$.
   (b) Project $b = \cos(x)^3$ into the column space of Q. (you should understand what is going on near the blue line under eq. 4 on page 233. This is really a least-squares fit). Does b equal its projection? What does this have to do with part (a) of this problem (there should really only be one reasonable interpretation of this question)?
   (c) Now project $b = \cos(x)^5$ onto the column space of Q. Does b equal its projection? If the answer is different from the previous part, why not?
8. Do problem 14 from 5.1.
9. Do problem 29 from 5.1.

18.06 Wisdom. Really understand the determinant and think about how it relates to rank and invertibility. Learn to stereotype a matrix by whether its determinant is 0 or not (what do these two situations mean?) Think about what the determinant is (and what it means) for special matrices we've seen, such as the identity matrix, permutation matrices, elimination matrices, Q (or Q^T). If you understand these things, you're on your way to cooking with oil.