

Problem Set 1, 18.06 Fall '11

This problem set is due Thursday, September 15, 2011 at 4pm. The problems are out of the 4th edition of the textbook. For computational problems, please include a printout of the code with the problem set (for MATLAB in particular, `diary("filename")` will start a transcript session, `diary off` will end one.)

- Do Problem 17 from 2.1. Treat the vectors as *column* vectors.
 - Calculate PQ . Calculate QP . Think about the significance of the answers (no explanation necessary).
- Do Problem 12 from 2.2.
- Do Problem 16 from 2.3.
- Recall that in Gauss-Jordan we took matrices $M = [A \ I]$, where I is the identity, and performed transformations to get $N = E_1 E_2 \cdots M = [I \ A^{-1}]$. Suppose you applied the same eliminations to the matrix $M = [A \ B]$ where A is the same as before but B is a more general matrix than the identity I . We should still get a matrix $[I \ X]$. What is X in terms of A and B ?
 - Write some code that, given $M = [A \ B]$, produces the matrix $[I \ X]$. *If you did the previous part correctly, you do NOT have to (or want to) do this with actual row operations, as long as you get the same result as if you did. The solution should be only a couple of lines.* (Hint for MATLAB users: the backslash is helpful).
 - Show that the code works (by attaching the output) for two matrices of block form $M = [A \ B]$ with different dimensions.
- Do Problem 10 from 2.5.
- Do Problem 18 from 2.5.
- For every $n = 1, 2, 3, \dots$, there is a very important matrix (with complex entries) known as the fourier matrix F_n . We will modify it slightly and work with $G_n = F_n/\sqrt{n}$. (In MATLAB it is obtained by `G_n=fft(eye(n))/sqrt(n)`. In other languages you can use some version of `FourierMatrix` or write loops to make the (i, j) -th entry

$$f(i, j) = \exp(-2ij\pi\sqrt{-1}/n)/\sqrt{n}$$

for $i, j = 1, \dots, n$)

- Find $(G_n)^4$, via a computer, for any 3 values of n . Make a conjecture for what $(G_n)^4$ is and do not try to prove it (the proof is outside the scope of the class, but there should be no doubt to the conjecture).
- Assuming your conjecture is true, what is $(G_n)^{-1}$ in terms of G_n ? Your expression should be a purely algebraic expression with no negative signs or words.

- (c) Let M be a random matrix of dimension n (in MATLAB this can be done by `M=rand(n)`). What is the relationship between $G_n M$ and $(G_n)^9 M$? The actual use of the computer is optional if you can justify your reasoning.
8. Do Problem 2.5 number 40... with a twist. This problem can be done: 1) by hand, 2) by guessing with a numerical computation, or 3) by a symbolic computation. Do the problem in 2 of these 3 ways. Some hints/help:

- If you are trying to be a good guesser, use $a = 2$, $b = 3$, and $c = 10$ and see how good a pattern detector you are. This can be done by hand (good practice!) or MATLAB, where you'd do something like
`A=eye(4)-diag([2 3 10],1)`
`inv(A)`.
- If you want to do symbolic computation in MATLAB, if the symbolic toolbox is available you can do
`syms a b c`
`A=eye(4)-diag([a b c],1)`.
- If you want to do symbolic computation in Mathematica, you can do
`A={{1,-a,0,0},{0,1,-b,0},{0,0,1,-c},{0,0,0,1}}`
`MatrixForm[Inverse[A]]`.

18.06 Wisdom. “The kind of skill that makes you a good guesser is valuable in life. Even better is to develop the skill that makes you think that 2, 3, and 10 are good guesses.” - Professor Edelman