18.06 Professor Postnikov Quiz 3 November 23, 2009

Your PRINTED name is: ____________________

Please circle your recitation:

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(R02)	T11	2-132	HwanChul Yoo
(R03)	T12	2-132	David Shirokoff
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(R08)	T3	2-146	Steven Sivek

Grading	
1	
2	
3	
Total:	

Problem 1. Let $A = \begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix}$.

(A) Find the eigenvalues and the eigenvectors of A.

(B) Solve the differential equation $\dfrac{d\,\mathbf{u}(t)}{dt} = A\,\mathbf{u}(t)$ with the initial condition $\mathbf{u}(0) = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$.

(C) Find a symmetric matrix B which is similar to A.

(D) Find the singular values σ_1 and σ_2 of A.

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Problem 2. Consider the matrix

$$A = \begin{pmatrix} 1 & t & 0 \\ t & 1 & 1 \\ 0 & 1 & 2 \end{pmatrix},$$

which depends on a parameter t.

(A) Find all values of the parameter t when the matrix A is positive definite.

(B) Suppose that $t = 0$. Find a 3×3 matrix R such that $A = R^T R$.

(C) Suppose that $t = 0$. Verify directly that A satisfies the energy-based definition of a positive definite matrix, as follows. For a vector $\mathbf{x} = (x, y, z)^T$, write out $\mathbf{x}^T A\,\mathbf{x}$; show that this can be written as a sum of squares; and deduce that $\mathbf{x}^T A\,\mathbf{x} > 0$ for any non-zero $\mathbf{x}$.

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Problem 3. Let $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 0 \\ 1 & 0 & 2 \end{pmatrix}$.

(A) Indicate which of the following statements are true and which are false:

(1) A is symmetric; (2) A is orthogonal;

(3) A is invertible; (4) $\frac{1}{3}\,A$ is a Markov matrix

(B) Find the eigenvalues and the eigenvectors of A. (Hint: Part (A) might help you.)

(C) Find an orthogonal matrix Q and a diagonal matrix Λ such that $A = Q\Lambda Q^T$.

(D) Calculate the limit $\mathbf{u}_\infty$ of $\mathbf{u}_k = (\frac{1}{3}\,A)^k\,\mathbf{u}_0$ as $k \to \infty$, for $\mathbf{u}_0 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$.

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