18.06 Professor Postnikov Quiz 1 October 5, 2009

Your PRINTED name is: ____________________

Please circle your recitation:

(R01)	T10	2-132	HwanChul Yoo
(R02)	T11	2-132	HwanChul Yoo
(R03)	T12	2-132	David Shirokoff
(R04)	T1	2-131	Fucheng Tan
(R05)	T1	2-132	David Shirokoff
(R06)	T2	2-131	Fucheng Tan
(R07)	T2	2-146	Leonid Chindelevitch
(R08)	T3	2-146	Steven Sivek

Grading
1
2
3
4
Total:

Problem 1. Consider the matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \\ 0 & 1 & 3 \end{pmatrix}$.

(a) Find the factorization $A = LU$.

(b) Find the inverse of A.

(c) For which values of c is the matrix $\begin{pmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \\ 0 & 1 & c \end{pmatrix}$ invertible?

Problem 2. Which of the following are subspaces? Explain why.

(a) All vectors $\mathbf{x}$ in $\mathbb{R}^3$ such that $\mathbf{x}^T \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = 0$.

(b) All vectors $(x, y)^T$ in $\mathbb{R}^2$ such that $x^2 - y^2 = 0$.

(c) All vectors $(x, y)^T$ in $\mathbb{R}^2$ such that $x + y = 2$.

(d) All vectors $\mathbf{x}$ in $\mathbb{R}^3$ which are in the column space AND in the nullspace of the matrix $\begin{pmatrix} 1 & -2 & 1 \\ 1 & -2 & 1 \\ 1 & -2 & 1 \end{pmatrix}$.

(e) All vectors $\mathbf{x}$ in $\mathbb{R}^3$ which are in the column space OR in the nullspace (or in both) of the matrix $\begin{pmatrix} 1 & -2 & 1 \\ 1 & -2 & 1 \\ 1 & -2 & 1 \end{pmatrix}$.

Problem 3. Consider the matrix

$$A = \begin{pmatrix} 1 & 2 & 1 & 2 & 2 \\ -1 & -2 & 0 & 0 & -1 \\ 1 & 2 & 0 & 0 & 1 \end{pmatrix}$$

(a) Find the complete solution of the equation $A\mathbf{x} = \mathbf{0}$.

(b) Find the complete solution of the equation $A\mathbf{x} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$.

(c) Find all vectors $\mathbf{b}$ such that the equation $A\mathbf{x} = \mathbf{b}$ has a solution.

(d) Find a matrix B such that $N(A) = C(B)$.

(e) Find bases of the four fundamental subspaces for the matrix A.

Problem 4. Let A be an m by n matrix. Let B be an n by m matrix. Suppose that $AB = I_m$ is the m by m identity matrix.

1. Let $r = \text{rank}(A)$ denote the rank of the matrix A. Choose one answer and be sure to justify it.

   (a) $r \geq m$

   (b) $r \leq m$

   (c) $r = m$

   (d) $r > n$

2. Is $m \leq n$ or is $n \leq m$? Why?