|  | Grading |
| :--- | :--- |
| Your PRINTED name is: | 1 |
|  | 2 <br> 3 <br> Please circle your recitation: |


| 1) | T 10 | 2-131 | J.Yu | 2-348 | 4-2597 | jyu |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2) | T 10 | 2-132 | J. Aristoff | 2-492 | 3-4093 | jeffa |
| 3) | T 10 | 2-255 | Su Ho Oh | 2-333 | 3-7826 | suho |
| 4) | T 11 | 2-131 | J. Yu | 2-348 | 4-2597 | jyu |
| 5) | T 11 | 2-132 | J. Pascaleff | 2-492 | 3-4093 | jpascale |
| 6) | T 12 | 2-132 | J. Pascaleff | 2-492 | 3-4093 | jpascale |
| 7) | T 12 | 2-131 | K. Jung | 2-331 | 3-5029 | kmjung |
| 8) | T 1 | 2-131 | K. Jung | 2-331 | 3-5029 | kmjung |
| 9) | T 1 | 2-136 | V. Sohinger | 2-310 | 4-1231 | vedran |
| 10) | T 1 | 2-147 | M Frankland | 2-090 | 3-6293 | franklan |
| 11) | T 2 | 2-131 | J. French | 2-489 | 3-4086 | jfrench |
| 12) | T 2 | 2-147 | M. Frankland | 2-090 | 3-6293 | franklan |
| 13) | T 2 | 4-159 | C. Dodd | 2-492 | 3-4093 | cdodd |
| 14) | T 3 | 2-131 | J. French | 2-489 | 3-4086 | jfrench |
| 15) | T 3 | 4-159 | C. Dodd | 2-492 | 3-4093 | cdodd |

1 (30 pts.) The complex matrix

$$
A=\left[\begin{array}{cc}
a & c+d i \\
c-d i & b
\end{array}\right]
$$

where $a, b, c$, and $d \neq 0$ are real numbers.
In (a) and (b) below circle the one best answer to the questions:
(a) This matrix is necessarily: symmetric? Hermitian? unitary? Markov?
(b) The two eigenvalues are necessarily: real? positive? zero? complex conjugates?
(c) The sum of the two eigenvalues is
(d) The product of the two eigenvalues in terms of $a, b, c$, and $d$ but not $i$ is $\qquad$
(e) In terms of an eigenvalue $\lambda$ (whose value you need not derive), write down an eigenvector of $A$.

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2 (32 pts.) The real matrix

$$
A=\left[\begin{array}{cc}
x & 3 / 5 \\
y & z
\end{array}\right]
$$

The answers to the questions below involve alternative equations or inequalities involving $x, y$, and $z$ that characterize all matrices of a certain type. Write down the relations. For (a) through (c), credit is only given for the complete description in reasonably clear and simple form.
(a) When is $A$ positive definite? (Write two inequalities.)
(b) When is $A$ Markov? (Perhaps write two or more inequalities, and two equalities.)
(c) When is $A$ singular? (Write one equality)
(d) Write down one such $A$ that is orthogonal. (There are four possible $A$ and you are asked to write down one.)

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3 (13 pts.) The 4 x 4 Fourier matrix $F$ has eigenvalues $-2,2,2 i,-2 i$. Preferably without any explicit computation ( or even knowledge of the matrix itself) what is the matrix $F^{4}$ ? How do you know it has that particular Jordan form?

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4 (25 pts.) In terms of $x(0<x<1)$ complete

$$
A=\left[\begin{array}{ll}
x & \\
&
\end{array}\right]
$$

so that $A$ is a $2 \times 2$ matrix that is both Markov and singular.
What is $A^{2008} ?$

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