18.06 Professor Edelman Quiz 2 October 22, 2008 $\begin{array}{ll} & \text { Grading } \\ \text { Your PRINTED name is: } & 1 \\ & \begin{array}{l}1 \\ 2 \\ \text { Please circle your recitation: }\end{array} \\ 3 \\ 4 \\ & 5\end{array}$

| 1) | T 10 | 2-131 | J.Yu | 2-348 | 4-2597 | jy |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2) | T 10 | 2-132 | J. Aristoff | 2-492 | 3-4093 | jeffa |
| 3) | T 10 | 2-255 | Su Ho Oh | 2-333 | 3-7826 | suho |
| 4) | T 11 | 2-131 | J. Yu | 2-348 | 4-2597 | jyu |
| 5) | T 11 | 2-132 | J. Pascaleff | 2-492 | 3-4093 | jpascale |
| 6) | T 12 | 2-132 | J. Pascaleff | 2-492 | 3-4093 | jpascal |
| 7) | T 12 | 2-131 | K. Jung | 2-331 | 3-5029 | kmjung |
| 8) | T 1 | 2-131 | K. Jung | 2-331 | 3-5029 | kmjung |
| 9) | T 1 | 2-136 | V. Sohinger | 2-310 | 4-1231 | vedran |
| 10) | T 1 | 2-147 | M Frankland | 2-090 | 3-6293 | franklan |
| 11) | T 2 | 2-131 | J. French | 2-489 | 3-4086 | jfrench |
| 12) | T 2 | 2-147 | M. Frankland | 2-090 | 3-6293 | franklan |
| 13) | T 2 | 4-159 | C. Dodd | 2-492 | 3-4093 | cdodd |
| 14) | T 3 | 2-131 | J. French | 2-489 | 3-4086 | jfrench |
| 15) | T 3 | 4-159 | C. Dodd | 2-492 | 3-4093 | cdodd |

1 ( $\mathbf{1 0}$ pts.) The determinant of the 1000 by 1000 matrix $A$ is 12 . What is the determinant of $(-A)^{T}$ ? (Careful: No credit for the wrong sign.)

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(a) $P$ is the projection matrix onto the column space of $A$ which has independent columns. $Q$ is a square orthogonal matrix with the same number of rows as $A$. In simplest form, in terms of $P$ and $Q$, what is the projection matrix onto the column space of $Q A$ ?
(b) The vectors $a, b$, and $c$ are independent. The matrix $P$ is the projection matrix onto the span of $a$ and $b$. Suppose we apply Gram-Schmidt onto the vectors $a, b$, and $c$ producing orthonormal vectors $q_{1}, q_{2}$, and $q_{3}$. Write the unit vector $q_{3}$ in simplest form in terms of $P$ and $c$ only.
(c) The vector $a, b$ and $c$ are independent. The matrix $A=\left[\begin{array}{ll}a & b c\end{array}\right]$ has these three vectors as its columns. The QR decomposition writes $A=Q R$ where $Q$ is orthogonal and $R$ is $3 \times 3$ upper triangular. Write $\|c\|$ in terms of only the elements of $R$ in simplest form.

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3 ( $\mathbf{1 5}$ pts.) The vector $u$ is a "unit vector" meaning $\|u\|=1$. What are all the possible values of $t$ which guarantee that the matrix $A=I+t u u^{T}$ is orthogonal?

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4 (15 pts.) Suppose we have obtained from measurements $n$ data points $\left(t_{i}, b_{i}\right)$, and you are asked to find a best least squares fit function of the form $y=$ $C+D t+E(1-t)$. Are $C, D$, and $E$ uniquely determined? Write down a solvable system of equations that gives a solution to the least squares problem.

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5 ( $\mathbf{3 0}$ pts.) (a) If $A$ is invertible, must the column space of $A^{-1}$ be the same as the column space of $A$ ?
(b) If $A$ is square, must the column space of $A^{2}$ be the same as the column space of $A$ ?

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