18.06 Professor Johnson Quiz 3 December 3, 2007

Your PRINTED name is: ____________

Please circle your recitation:

(1)	T 10	2-131	B. Mares
(2)	T 10	2-132	A. Barakat
(3)	T 11	2-132	A. Barakat
(4)	T 11	2-131	B. Lehmann
(5)	T 12	2-132	A. Spiridonov
(6)	T 12	2-131	B. Lehmann
(7)	T 1	2-131	A. Spiridonov
(8)	T 2	2-131	Y. Lekili
(9)	T 2	4-159	Z. Wang
(10)	T 3	2-131	Y. Lekili

Grading	
1	
2	
3	
4	
Total:	

1 (20 pts.) True or false. Explain why if *false*, or give an example if *true*.

(a) There exist matrices $A \neq 0$ that are simultaneously Hermitian ($A = A^H$) and unitary ($A^H = A^{-1}$).

(b) There exist matrices $A \neq 0$ that are simultaneously anti-Hermitian ($A = -A^H$) and unitary ($A^H = A^{-1}$).

(c) There exist matrices $A \neq 0$ that are simultaneously Hermitian ($A = A^H$) and anti-Hermitian ($A = -A^H$).

(d) There exist matrices A that are simultaneously Hermitian and Markov.

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2 (30 pts.) Suppose we form a sequence of real numbers f_k defined by the recurrence $f_{k+1} = f_k - f_{k-1} + f_{k-2}$, starting with the initial conditions $f_0 = 2$, $f_1 = 1$ and $f_2 = 0$.

(a) Define a 3-component vector $\vec{g}_k = (f_k, f_{k-1}, f_{k-2})^T$ and a 3×3 matrix A so that the recurrence is $\vec{g}_{k+1} = A\vec{g}_k$.

(b) If you constructed A correctly, the three eigenvalues should be 1 and $\pm i$ [I'm giving you these so you *don't* have to solve a cubic equation], and the latter two eigenvectors should be $(-1, \pm i, 1)^T$. Check that you have these $\pm i$ eigenvalues and eigenvectors, and find the $\lambda = 1$ eigenvector.

(c) Give an explicit formula for f_k for any k. (By "explicit," I mean involving elementary arithmetic and powers of complex numbers only. Formulas involving A^k are not acceptable.)

(d) Is there any choice of initial conditions that will make $|f_k|$ diverge as $k \to \infty$? Explain.

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3 (30 pts.)

(a) Suppose $A = e^{iB}$ where B is Hermitian; what is $A^H A$? Hence A is a ______________ matrix.

(b) For the recurrence relation $\vec{f}_{k+1} = e^{iB}\vec{f}_k$, what is $\|\vec{f}_k\|^2/\|\vec{f}_0\|^2$? [Hint: part (a) is useful.]

(c) Compute $\vec{f}_k$ explicitly [i.e. no matrix exponentials or powers of matrices] for $B = \begin{pmatrix} -3 & 4 \\ 4 & 3 \end{pmatrix}$ and $\vec{f}_0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$. The eigenvectors of this B are $\vec{x}_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $\vec{x}_2 = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ with eigenvalues $\lambda_1 = 5$ and $\lambda_2 = -5$, respectively.

(d) Check that your answer from (b) is true for your answer from (c).

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4 (20 pts.) Some 3×3 real matrix A has eigenvalues $\lambda_1 = 0$, $\lambda_2 = 1$, and $\lambda_3 = 2$, with the corresponding eigenvectors $\vec{x}_1 = (1,0,0)^T$, $\vec{x}_2 = (0,1,2)^T$, and $\vec{x}_3 = (0,1,1)^T$.

(a) Give a basis for: (i) the nullspace $N(A)$, (ii) the column space $C(A)$, and (iii) the row space $C(A^H)$.

(b) Find all solutions $\vec{x}$ to $A\vec{x} = \vec{x}_2 - 3\vec{x}_3$.

(c) Is A (i) real-symmetric, (ii) orthogonal, (iii) Markov, or (iv) none of the above?

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