18.06 Professor Johnson Quiz 2 October 31, 2007

Your PRINTED name is: ____________

Please circle your recitation:

(1)	T 10	2-131	B. Mares
(2)	T 10	2-132	A. Barakat
(3)	T 11	2-132	A. Barakat
(4)	T 11	2-131	B. Lehmann
(5)	T 12	2-132	A. Spiridonov
(6)	T 12	2-131	B. Lehmann
(7)	T 1	2-131	A. Spiridonov
(8)	T 2	2-131	Y. Lekili
(9)	T 2	4-159	Z. Wang
(10)	T 3	2-131	Y. Lekili

Grading
1
2
3
4
5
Total:

1 (20 pts.) Let $\mathbf{v} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$.

(a) Show that $A = \mathbf{v}\mathbf{v}^T$ is a symmetric 3×3 matrix. What is the rank of A?

(b) Find the projection matrix P onto the **nullspace** of A.

(c) Calculate PA and AP.

(d) Show that for any symmetrix $n \times n$ matrix B, and the projection matrix Q onto the nullspace of G, we have $QA = AQ = 0$.

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2 (20 pts.) Given 4 points $(t, b) = (0, 1), (1, 2), (2, 2)$ and $(3, 3)$.

(a) Use the least square methods to find a line $b = C + Dt$ which best fits the four points.

(b) Find equations (**do not solve**) for the coefficients C', D', E' in $b = C' + D't + E't^2$, the parabola which best fits the four points above.

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3 (16 pts.) Fill in the blanks below.

(a) The nullspace of AB contains the nullspace of ____________.

(b) Let P be the projection matrix to the column space of a matrix A, then $I - P$ is the projection matrix to ____________.

(c) The projection of the vector $\begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}$ onto the nullspace of the matrix $\begin{pmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 4 \end{pmatrix}$ is ____________.

(d) Suppose A is an $m \times n$ matrix, and the row space of A is n dimensional, then its nullspace is ____________ dimensional.

(e) Let $\hat{\mathbf{x}}$ be the least-squares solution to $A\mathbf{x} = \mathbf{b}$. Then $\mathbf{b} - A\hat{\mathbf{x}}$ is orthogonal to the ____________ of A.

(f) Let A be the $m \times n$ (edge-node) incidence matrix of a connected graph with n vertices and m edges. Then the left nullspace of A has dimension ____________.

(g) Suppose A is a 5×5 matrix, and $\det(A) = 2$. Then $\det(2AA^T) =$ ____________.

(h) If A is an arbitrary orthogonal matrix, the possible values of $\det(A)$ are ____________.

4 (16 pts.) (a) Compute the determinant of the matrix $\begin{pmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{pmatrix}$.

(b) Suppose A is any antisymmetric $n \times n$ matrix, and n is odd. Find the determinant of A.

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5 (28 pts.) Let $A = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 2 & 2 \\ 0 & 3 & 3 & 3 \\ 4 & 4 & 4 & 4 \end{pmatrix}$.

(a) Do Gram-Schemidt, but without normalizing lengths, to get an orthogonal but not orthonormal basis B for $C(A)$.

(b) Compute the (4-dimensional) volume of the 2 parallelepipeds with edges given by the columns of A and the columns of B.

(c) For an arbitrary $n \times n$ matrix A, write down this "unnormalized" Gram-Schmidt process in terms of A multiplied by a sequence of matrices (similar to the QR decomposition), and use it to prove that this process perserves the volumn.

(d) Show that for any orthogonal matrix B, $B^T B$ is a diagonal matrix, then use this to derive that $\det(B)$ equals the product of the lengths of its column vectors.

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