18.06 Professor Johnson Quiz 1 October 3, 2007

Your PRINTED name is: ____________

Please circle your recitation:

(1)	T 10	2-131	B. Mares
(2)	T 10	2-132	A. Barakat
(3)	T 11	2-132	A. Barakat
(4)	T 11	2-131	B. Lehmann
(5)	T 12	2-132	A. Spiridonov
(6)	T 12	2-131	B. Lehmann
(7)	T 1	2-131	A. Spiridonov
(8)	T 2	2-131	Y. Lekili
(9)	T 2	4-159	Z. Wang
(10)	T 3	2-131	Y. Lekili

Grading

1
2
3
4
Total:

1 (20 pts.) Find all solutions to the linear system

$$x + 2y + z - 2w = 5$$
$$2x + 4y + z + w = 9$$
$$3x + 6y + 2z - w = 14$$

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2 (30 pts.) In class, we learned how to do "downwards" elimination to put a matrix A in upper-triangular (or echelon) form U: not counting row swaps, we subtracted multiples of pivot rows from subsequent rows to put zeros below the pivots, corresponding to multiplying A by elimination matrices.

Instead, we could do elimination "leftwards" by subtracting multiples of pivot columns from leftwards columns, again to get an upper-triangular matrix U. For example, let:

$$A = \begin{pmatrix} 7 & 6 & 4 \\ 6 & 3 & 12 \\ 2 & 0 & 1 \end{pmatrix}$$

We could subtract twice the third column from the first column to eliminate the 2, so that we get zeros to the left of the "pivot" 1 at the lower right.

(i) Continue this "leftwards" elimination to obtain an upper-triangular matrix U from the A above, and write U in terms of A multiplied by a sequence of matrices corresponding to each leftwards-elimination step.

(ii) Suppose we followed this process for an arbitrary A (not necessarily square or invertible) to get an echelon matrix U. Which of the column space and null space, if any, are the same between A and U, and why?

(iii) Is the U that we get by leftwards elimination always the same as the U we get from ordinary downwards elimination? Why or why not?

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3 (20 pts.) Determine whether the following statements are true or false, and explain your reasoning.

(♣) If $A^2 = A$, then $A = 0$ or $A = I$.

(♢) Ignoring row swaps, any invertible matrix A has a "UL" factorization (as an alternative to LU factorization): A can be written as $A = UL$ where U and L are some upper and lower triangular matrices, respectively.

(♠) All the 2×2 matrices that commute with $A = \begin{pmatrix} 1 & 3 \\ 2 & 0 \end{pmatrix}$ (i.e. all 2×2 matrices B with $AB = BA$) form a vector space (with the usual formulas for addition of matrices and multiplication of matrices by numbers).

(♡) There is no 3×3 matrix whose column space equals its nullspace.

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4 (30 pts.) The following information is known about an $m \times n$ matrix A:

$$A\begin{pmatrix}1\\-2\\0\\1\end{pmatrix}=\begin{pmatrix}2\\4\end{pmatrix},\ A\begin{pmatrix}0\\2\\1\\3\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix},\ A\begin{pmatrix}2\\0\\0\\1\end{pmatrix}=\begin{pmatrix}5\\10\end{pmatrix},\ A\begin{pmatrix}3\\2\\0\\0\end{pmatrix}=\begin{pmatrix}1\\2\end{pmatrix}.$$

(α) Show that the vectors $\begin{pmatrix}1\\-2\\0\\1\end{pmatrix},\begin{pmatrix}0\\2\\1\\3\end{pmatrix},\begin{pmatrix}2\\0\\0\\1\end{pmatrix},\begin{pmatrix}3\\2\\0\\0\end{pmatrix}$ form a basis of $\mathbb{R}^4$.

(β) Give a matrix C and an invertible matrix B such that $A = CB^{-1}$. (You *don't* have to evaluate B^{-1} or find A explicitly. Just say what B and C are and use them to reason about A in the subsequent parts.)

(γ) Find a basis for the null space of A^T.

(δ) What are m, n, and the rank r of A?

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