18.06 Professor Strang Quiz 3 December 4, 2006

Grading

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Your PRINTED name is: ____________________

Please circle your recitation:

1)	T 10	2-131	K. Meszaros	2-333	3-7826	`karola`
2)	T 10	2-132	A. Barakat	2-172	3-4470	`barakat`
3)	T 11	2-132	A. Barakat	2-172	3-4470	`barakat`
4)	T 11	2-131	A. Osorno	2-229	3-1589	`aosorno`
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6)	T 12	2-131	K. Meszaros	2-333	3-7826	`karola`
7)	T 1	2-132	A. Edelman	2-343	3-7770	`edelman`
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9)	T 3	2-132	A. Osorno	2-229	3-1589	`aosorno`

1 (34 pts.) (a) If a square matrix A has all n of its *singular values* equal to 1 in the SVD, what basic classes of matrices does A belong to? (Singular, symmetric, orthogonal, positive definite or semidefinite, diagonal)

(b) Suppose the (orthonormal) columns of H are eigenvectors of B:

$$H = \frac{1}{2}\begin{bmatrix} 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix} \qquad H^{-1} = H^{\mathrm{T}}$$

The eigenvalues of B are $\lambda = 0, 1, 2, 3$. Write B as the product of 3 specific matrices. Write $C = (B + I)^{-1}$ as the product of 3 matrices.

(c) Using the list in question (a), which basic classes of matrices do B and C belong to? (Separate question for B and C)

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2 (33 pts.) (a) Find three eigenvalues of A, and an eigenvector matrix S:

$$A = \begin{bmatrix} -1 & 2 & 4 \\ 0 & 0 & 5 \\ 0 & 0 & 1 \end{bmatrix}$$

(b) Explain why $A^{1001} = A$. Is $A^{1000} = I$? Find the three diagonal entries of e^{At}.

(c) The matrix $A^{\mathrm{T}}A$ (for the same A) is

$$A^{\mathrm{T}}A = \begin{bmatrix} 1 & -2 & -4 \\ -2 & 4 & 8 \\ -4 & 8 & 42 \end{bmatrix}.$$

How many eigenvalues of $A^{\mathrm{T}}A$ are positive? zero? negative? (Don't compute them but explain your answer.) Does $A^{\mathrm{T}}A$ have the same eigenvectors as A?

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3 (33 pts.) Suppose the n by n matrix A has n orthonormal eigenvectors $q_1, \ldots, q_n$ and n positive eigenvalues $\lambda_1, \ldots, \lambda_n$. Thus $Aq_j = \lambda_j q_j$.

(a) What are the eigenvalues and eigenvectors of A^{-1}? *Prove that your answer is correct.*

(b) Any vector b is a combination of the eigenvectors:

$$b = c_1 q_1 + c_2 q_2 + \cdots + c_n q_n \,.$$

What is a quick formula for c_1 using orthogonality of the q's?

(c) The solution to $Ax = b$ is also a combination of the eigenvectors:

$$A^{-1} b = d_1 q_1 + d_2 q_2 + \cdots + d_n q_n \,.$$

What is a quick formula for d_1? You can use the c's even if you didn't answer part (b).

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