

## 18.06 Problem Set 8

Due Wednesday, Nov. 15, 2006 at **4:00 p.m.** in 2-106

### Problem 1 Monday 11/6

Do Problem #12 from section 8.3 in your book.

### Problem 2 Monday 11/6

Of 300 million Americans, 60% own their own home and the other 40% rent.

Let's represent these proportions as a vector:  $x = \begin{bmatrix} \text{owners} \\ \text{renters} \end{bmatrix} = \begin{bmatrix} .60 \\ .40 \end{bmatrix}$ .

Every year, some proportion of renters will buy a house, and some proportion of homeowners will move to a rental. If these proportions remain constant, we can model this with the "Markov process"  $x_{k+1} = Ax_k$  for some 2-by-2 Markov matrix  $A$ .

Suppose the proportion of homeowners and renters is modeled by this Markov process;

it maintains the steady state  $x = \begin{bmatrix} .60 \\ .40 \end{bmatrix}$  above;

and 90 percent of homeowners in any given year  $k$  still own a home in year  $k + 1$ .

Determine  $A$ , and estimate how many American renters will buy a home this year.

### Problem 3 Wednesday 11/8

Find the first three nonzero terms in the Fourier series for the period- $2\pi$  function

$$f(t) = \begin{cases} 1, & 0 < t < \pi \\ 0, & \pi < t < 2\pi \end{cases}$$

Then find the lengths of the original function  $\|f(t)\|$  and your three-term approximation  $\|g(t)\|$ , and the distance  $\|f(t) - g(t)\|$  between them.

### Problem 4 Wednesday 11/8

Do Problem #1 from section 10.2 in your book.

### Problem 5 Wednesday 11/8

Do Problem #2 from section 10.2 in your book.

### Problem 6 Wednesday 11/8

Do Problem #17 from section 10.2 in your book.

### Problem 7 Wednesday 11/8

Do Problem #31 from section 10.2 in your book.

(Hints:  $U$  is a \_\_\_\_\_ matrix, so  $U^H U =$  \_\_\_\_\_.  $\Lambda$  is a \_\_\_\_\_ matrix, so  $\Lambda^H \Lambda$  and  $\Lambda \Lambda^H$  are \_\_\_\_\_.)

### Problem 8 Wednesday 11/8

Do Problem #7 from section 10.3 in your book.

**Problem 9** *Monday 11/13*

Do Problem #16 from section 6.3 in your book.

**Problem 10** *Monday 11/13*

Do Problem #22 from section 6.3 in your book.

Then solve  $u' = \begin{bmatrix} 1 & 1 \\ 0 & 3 \end{bmatrix} u$  for initial condition  $u(0) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ .

Is the solution stable as  $t \rightarrow \infty$ ? Why or why not?