18.06 Problem Set 5

Due Wednesday, Oct. 18, 2006 at **4:00 p.m.** in 2-106

Problem 1 *Wednesday 10/11*

For each of these, find a matrix satisfying the conditions given or explain why none can exist.

(a) Column space contains $\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$, and nullspace contains $\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$

(b) Row space contains $\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$, and nullspace contains $\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$

(c) $Ax = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ is solvable; $A^{\mathsf{T}} \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$ is zero.

(d) A *nonzero* matrix where every row is perpendicular to every column

(e) Rows sum to a row of zeros, and columns sum to $\begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$

Problem 2 *Wednesday 10/11*

Do Problem #12 from section 4.1 in your book.

Problem 3 *Wednesday 10/11*

Do Problem #26 from section 4.1 in your book.

Problem 4 *Friday 10/13*

Do Problem #2 from section 4.2 in your book. What is the permutation matrix P? What is the error $e = b - p$?

Problem 5 *Friday 10/13*

Do Problem #13 from section 4.2 in your book. Do this two different ways:
(a) geometrically, tell what subspace we're projecting b orthogonally onto
(b) algebraically, calculate $P = A(A^{\mathsf{T}}A)^{-1}A^{\mathsf{T}}$

Problem 6 *Friday 10/13*

A subspace **S** has basis $\left\{ a = \begin{bmatrix} 2 \\ -1 \\ 0 \\ 1 \end{bmatrix}, b = \begin{bmatrix} 5 \\ 0 \\ -1 \\ 2 \end{bmatrix}, c = \begin{bmatrix} 0 \\ 5 \\ 4 \\ -1 \end{bmatrix} \right\}$.

(a) What are the dot products $a^{\mathsf{T}}b$, $a^{\mathsf{T}}c$, $b^{\mathsf{T}}c$? Are the basis vectors orthogonal?

Now let's compute a new basis $\left\{ \hat{a}, \hat{b}, \hat{c} \right\}$ for the same subspace. Start by letting $\hat{a} = a$.

(b) Compute the projection Pb of b onto the line described by a. What is the error $(b - Pb)$? Call this error vector $\hat{b}$.

(c) Compute the projection P_1c of c onto the plane described by a and b. What is the error $(c - P_1c)$? Call this error vector $\hat{c}$. Does $\hat{c}$ change if we project onto the plane with basis $\hat{a}$ and $\hat{b}$ instead? Why or why not?

(d) What are the dot products $\hat{a}^\top \hat{b}$, $\hat{a}^\top \hat{c}$, $\hat{b}^\top \hat{c}$? Are the new basis vectors orthogonal?
(This process for finding an orthogonal basis is called the "Gram-Schmidt Process" — the full version also scales each vector to "normalize" it to unit length.)

(e) Find the matrix R relating the old basis and the new basis: $\begin{bmatrix} a & b & c \end{bmatrix} = \begin{bmatrix} \hat{a} & \hat{b} & \hat{c} \end{bmatrix} \begin{bmatrix} ? & ? & ? \\ ? & ? & ? \\ ? & ? & ? \end{bmatrix}$

(f) Explain how you know $\left\{\hat{a}, \hat{b}, \hat{c}\right\}$ is a basis for $\mathbf{S}$. (Don't forget to show it both spans the subspace, and is linearly independent!)

Problem 7 *Monday 10/16*

Do Problem #17 from section 4.3 in your book.

Problem 8 *Monday 10/16*

Do Problem #27 from section 4.3 in your book.