18.06 Problem Set 2

Due Wednesday, Sept. 20, 2006 at **4:00 p.m.** in 2-106

Problem 1 *Monday 9/11*

Do Problem #7 from section 2.5 in your book.

Problem 2 *Monday 9/11*

Do Problem #9 from section 2.5 in your book.

Problem 3 *Monday 9/11*

Do Problem #25 from section 2.5 in your book.

Problem 4 *Monday 9/11*

Compute, by Gauss-Jordan elimination, a formula for the inverse of the 2-by-2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. What assumptions did you need to make?

Problem 5 *Wednesday 9/13*

Do Problem #2 from section 2.6 in your book.

Problem 6 *Wednesday 9/13*

(a) Do Problem #13 from section 2.6 in your book.
(b) Now compute the LDU-factorization $A = LDU$. Why is $U = L^{\mathsf{T}}$?

Problem 7 *Wednesday 9/13*

Do Problem #21 from section 2.6 in your book.

Problem 8 *Wednesday 9/13*

Back in 18.02, you learned a different method for solving $Ax = b$ called Cramer's rule, which wrote the coefficients of x as a quotient of determinants. You also learned to solve the system by finding A^{-1}.
Let's compare elimination, `x=inv(A)*b`, and Cramer's rule using Matlab.[1]
(a) Try each of the three methods on a random matrix of size 100, then of size 200 and 300. (You may want to try several matrices of each size, and take an average to get a better estimate.) What do you conclude?

```
Some commands you may find useful:
>> A=rand(100);          % A is a random 100-by-100 matrix
```

[1] If you're not using Athena, you may need the teaching code `cramer.m`, available from the 18.06 Web page: `web.mit.edu/18.06/www`

```
>> b=rand(100,1);          % B is a random 100-by-1 column vector
>> tic; cramer(A,b); toc % Time the solution by Cramer's rule
>> tic; inv(A)*b; toc    % by matrix inversion
>> tic; A\b; toc         % by elimination
>> for i = 1:5
      ...
   end                   % do ... five times
```

(b) How much faster is elimination than Cramer or `inv(A)`? (You could also try larger sizes.)

Problem 9 *Wednesday 9/13*

Factor the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 0 & -1 \\ 6 & 2 & 1 \end{bmatrix}$ as $A = LU$. (*Timesaving hint:* Look at problem 8 from Problem Set 1.)

Problem 10 *Friday 9/15*

Suppose $A = LU$, where $L = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ and $U = \begin{bmatrix} 2 & -1 & 0 \\ 0 & 2 & -1 \\ 0 & 0 & 2 \end{bmatrix}$. Let $b = \begin{bmatrix} 0 \\ 1 \\ 4 \end{bmatrix}$.

(a) Solve $Lc = b$ by back-substitution, then solve $Ur = c$ the same way for r.
Why does $x = r$ satisfy $Ax = b$?

(b) Oops! I meant to write $PA = LU$, where L,U are as above and $P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$.

What vector solves $Ax = b$ now?

Problem 11 *Friday 9/15*

(a) When is the product of two symmetric matrices symmetric? Explain your answer.

Problem 12 *Friday 9/15*

Let $P = P_{13}P_{26}P_{36}P_{14}P_{43}$, where the P_{ij} are permutation matrices of order 6.

(a) What is $P\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{bmatrix}$?

(b) What is P?

Problem 13 *Friday 9/15*

(a) What happens if we multiply a matrix A by the elimination matrix E_{ji} on the *right*?

(b) Solve by elimination: $y^{\mathsf{T}} \begin{bmatrix} 2 & 1 & 0 \\ -1 & 2 & 1 \\ 0 & -1 & 2 \end{bmatrix} = [1 \quad -4 \quad 3]$.

(c) How could you convert this to a row-elimination problem?