# 18.06 Problem Set 1

Due Wednesday, Sept. 13, 2006 at **4:00 p.m.** in 2-106

**Problem 1** *Wednesday 9/06*

Go read the Worked Examples 2.1A and 2.1B (page 29).
(You don't have to hand anything in for this problem.)

**Problem 2** *Wednesday 9/06*

Write the product $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} \pi \\ e \end{bmatrix}$ in two ways:
(a) as dot products of the rows with the column vector
(b) as a linear combination of the columns.

**Problem 3** *Wednesday 9/06*

(a) What matrix $A$ takes $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ to $A\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ to $A\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 7 \end{bmatrix}$?
(b) What is $A\begin{bmatrix} 1 \\ 2 \end{bmatrix}$?

**Problem 4** *Wednesday 9/06*

Do Problem #25 from section 2.1 in your book.

**Problem 5** *Wednesday 9/06*

Let's practice using Matlab by multiplying a random pair of upper-triangular matrices. (Hint: you can type `diary` at the beginning of your session to save a transcript.)
We'll need two matrices. *First you pick one:* Let `A=[a b c; 0 d e; 0 0 f]`, where `a...f` are six of your favorite *nonzero* numbers. *Now let the computer pick one:* `B=rand(3,3)` gives us a random 3-by-3 matrix; we can zero out the extra coefficients one-by-one by typing e.g. `B(3,2)=0`, or all at once by keeping only the upper-triangular part `B=triu(B)`
*Now compute* `A*B` *and* `B*A`. What shape is this new matrix? Are $AB$ and $BA$ equal?

**Problem 6** *Friday 9/08*

(a) Write examples of systems $A\vec{x} = \vec{b}$ where $A$ is a 3-by-3 matrix and:

1. in the row picture, all three planes are parallel but distinct
2. all three planes are equal
3. the three planes meet in a common line
4. in the column picture, $\vec{b}$ is a linear combination of the first two columns of $A$.
5. $\vec{b}$ is not a linear combination of the columns of $A$.

(b) How many solutions for each of these? Describe the shape (point, line, ...) of each solution set.
(c) Reduce each by elimination (you need not back-substitute) and check your answer.
Circle the pivots.

**Problem 7** *Friday 9/08*

Do Problem #6 from section 2.2 in your book.

**Problem 8** *Friday 9/08*

Consider the system of equations

$$\begin{aligned} 2x + y + z &= -1 \\ x - z &= 0 \\ 6x + 2y + z &= -1 \end{aligned}$$

Solve this system. (Eliminate, then back-substitute.)
Circle the pivots as you find them.
Write down the elimination matrices $E_{12}$, $E_{13}$, $E_{23}$ you used.

**Problem 9** *Friday 9/08*

Do Problem #22 from section 2.2 in your book.

**Problem 10** *Monday 9/11*

Consider the matrices $A = \begin{bmatrix} 0 & 1 & 4 \\ -2 & 3 & 6 \\ 2 & -1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 2 \\ 0 & 6 \\ 1 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & -1 \\ 4 & 2 \\ 0 & 1 \end{bmatrix}$.
(a) Find $AB$ and $AC$.
(b) What happens?
(c) Why does this tell you $A$ is not invertible?

**Problem 11** *Monday 9/11*

Do Problem #35 from section 2.4 in your book.