1.(a) $2, \frac{3}{2}, \frac{4}{3}, \frac{5}{4}$

(b) $A = \begin{pmatrix} 1 & & & \\ \frac{1}{2} & 1 & & \\ \frac{1}{2} & \frac{1}{3} & 1 & \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & 1 \end{pmatrix} \begin{pmatrix} 2 & & & \\ & \frac{3}{2} & & \\ & & \frac{4}{3} & \\ & & & \frac{5}{4} \end{pmatrix} \begin{pmatrix} 1 & & & \\ \frac{1}{2} & 1 & & \\ \frac{1}{2} & \frac{1}{3} & 1 & \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & 1 \end{pmatrix}^T$

(c) $c = -1/5$.since $E^2 = 4E$ and $AA^{-1} = (I+E)(I+cE) = I+(c+1+4c)E$ so $5c+1 = 0$.

2.(a) $P = \frac{aa^T}{a^Ta} = \frac{1}{7}\begin{pmatrix} 4 & 2 & 2 & 2 \\ 2 & 1 & 1 & 1 \\ 2 & 1 & 1 & 1 \\ 2 & 1 & 1 & 1 \end{pmatrix}$, Trick: computation easiest using $a = (2,1,1,1)^T$.

(b) The nullspace of $I - P$ consists of all multiples of $a$. (One view is that $x = Px$. Another view is that it is the orthogonal complement of the nullspace of $P$ which are all vectors orthogonal to $a$.)

(c) The matrix has rank 1 and is a projector, so one eigenvalue is 1 and the rest are 0.

3.(a) $E$ is rank one symmetric with trace 4, so the eigenvalues are $0, 0, 0, 4$.

(b) The eigenvalues of $A$ are $1, 1, 1$, and $1 + 2 \cdot 4 = 9$ so the determinant is 9.

(c) We need $1 + 4b > 0$ so $b > -1/4$.

4.(a) The columns of $B$ are independent since the columns of $A$ are. Therefore the span of $B$ is five dimensional. ($B$ has no free columns.)

(b) The nullspace of $B^T$ is a three dimensional subspace of $R^8$.It is the orthogonal complement of the column space of $B$ in $R^8$. The rref of $B$ looks like the first five columns of $I_8$.

(c) $B^TB$ is a $5 \times 5$ matrix with positive eigenvalues that are the squares of the singular values of $B$. $BB^T$ has the same five positive eigenvalues and three more 0 eigenvalues as well.

5.(a) The columns of $Q$ are $n$ orthonormal vectors in $R^m$.

(b) $n \leq m$ and the rank of $Q$ is $n$.

(c) $\hat{x} = Q^T b$.

6.(a) $1, i, -1, -i$

(b) The four columns of the DFT matrix $F_4 = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -i & -1 & i \\ 1 & -1 & 1 & -1 \\ 1 & i & 1 & -i \end{pmatrix}$

(c) diagonalizable-yes, permutation-yes, nonsingular-yes, Jordan-no, orthogonal-yes, projection-no, skew-symmetric-no

7.(a) The nullspace of $A$ has basis $v_2$ and $v_3$.

(b) $x = v_1/4 + c_1 v_2 + c_2 v_3$

(c) Shortest is $v_1/4$. It is the projection of $x$ above onto the span of $v_1$.

8.(a) $u_k = c_1 \lambda_1^k x_1 + c_2 \lambda_2^k x_2 + c_3 \lambda_3^k x_3$ and $u(t) = c_1 e^{\lambda_1 t} x_1 + c_2 e^{\lambda_2 t} x_2 + c_3 e^{\lambda_3 t} x_3$

(b)$\lambda_1 = 1$ and $|\lambda_i| < 1$, for $i = 2, 3$.

(c) The matrix may not have a complete set of independent eigenveectors. An example is a three by three Jordan block: $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$.