18.06 Fall 2004 Quiz 1 October 13, 2004

Your name is:

Please circle your recitation:

1. M2 A. Brooke-Taylor
2. M2 F. Liu
3. M3 A. Brooke-Taylor
4. T10 K. Cheung
5. T10 Y. Rubinstein
6. T11 K. Cheung
7. T11 V. Angeltveit
8. T12 V. Angeltveit
9. T12 F. Rochon
10. T1 L. Williams
11. T1 K. Cheung
12. T2 T. Gerhardt

Grading:

Question	Points	Maximum
Name + rec		5
1		15
2		55
3		25
Total:		100

Remarks:
Do all your work on these pages.
No calculators or notes.
Putting your name and recitation section correctly is worth 5 points.
The exam is worth a total of 100 points.

1. Let

$$A = \begin{bmatrix} 2 & 2 & 2 \\ 4 & 3 & 1 \\ -2 & -1 & 4 \end{bmatrix}.$$

(a) Compute an LDU factorization of A if one exists.

(b) Give all solutions to $Ax = b$ where $b = \begin{bmatrix} 2 \\ -3 \\ 11 \end{bmatrix}$.

2. **One of the entries of A has been modified as there was a mistake.** (Many of the subquestions are independent and can be answered in any order.) By performing row eliminations (and possibly permutations) on the following 4×8 matrix A

$$\begin{bmatrix} 1 & 2 & 0 & 3 & -1 & 1 & 1 & -2 \\ -3 & -6 & 2 & -7 & 7 & 0 & -6 & 3 \\ 1 & 2 & 2 & 5 & 3 & 3 & -1 & 0 \\ 2 & 4 & 0 & 6 & -2 & 1 & 3 & 0 \end{bmatrix}$$

we got the following matrix B:

$$\begin{bmatrix} 1 & 2 & 0 & 3 & -1 & 0 & 2 & 0 \\ 0 & 0 & 1 & 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(a) What is the rank of A?

(b) What are the dimensions of the 4 fundamental subspaces?

(c) How many solutions does $Ax = b$ have? Does it depend on b? Justify

(d) Are the rows of A linearly independent? Why?

(e) Do columns 4, 5, 6 and 7 of A form a basis of R^4? Why?

(f) Give a basis of $N(A)$.

(g) Give a basis of $N(A^T)$.

(h) (You do not need to do any calculations to answer this question.) What is the reduced row echelon form for A^T? Explain.

(i) (Again calculations are not necessary for this part.) Let $B = EA$. Is E invertible? If so, what is the inverse of E?

3. For each of these statements, say whether the claim is true or false and give a brief justification.

   (a) **True/False:** The set of 3×3 non-invertible matrices forms a subspace of the set of all 3×3 matrices.

   (b) **True/False:** If the system $Ax = b$ has no solution then A does not have full row rank.

(c) **True/False:** There exist $n \times n$ matrices A and B such that B is not invertible but AB is invertible.

(d) **True/False:** For any permutation matrix P, we have that $P^2 = I$.