18.06, Fall 2004, Problem Set 2

Due before 4PM on Wednesday September 22nd, 2004, in the boxes in 2-106. No late homework will be accepted. Don't forget to write your name and recitation section on the problem set. There is one box for each recitation section. For full credit, please be sure to show and explain your work. Exercises refer to the 3rd edition of the textbook.

Reading assignment: Remainder of chapter 2.

1. (a) Consider the elimination matrices $E_{21} = \begin{bmatrix} 1 & 0 & 0 \\ 5 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $E_{31} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -7 & 0 & 1 \end{bmatrix}$, $E_{32} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 3 & 1 \end{bmatrix}$. What are their inverses?

   (b) Compute the inverse of the matrix $M = E_{32}E_{31}E_{21}$? Do not compute what M is.

2. Let A be any $n \times n$ matrix.

   (a) If A is invertible, show that A^2 is invertible.

   (b) Suppose A^2 is invertible. Can A be singular? If yes, give such an example. If no, explain why A is non-singular.

3. Let
$$A = \begin{bmatrix} 3 & 2 & 6 \\ 6 & 4 & 11 \\ 3 & 3 & -8 \end{bmatrix}.$$
   Compute an LU factorization of A if one exists ($A = LU$), or express $PA = LU$ for some suitable permutation matrix P otherwise.

4. For which values of c is the following matrix non invertible? Justify.
$$A = \begin{bmatrix} 5 & c & c \\ c & c & c \\ 1 & 2 & c \end{bmatrix}.$$

5. Let $A = P_{25}P_{13}P_{16}P_{46}P_{36}$, where the P_{ij}'s are 6×6 permutation matrices. What is A?

6. (a) Is the inverse of a lower triangular matrix lower triangular also?

   (b) You will prove here that if there exists an LDU factorization then it is unique. Let's start the proof together. Assume $A = L_1D_1U_1 = L_2D_2U_2$, where L_1 and L_2 are lower triangular matrices with 1's on the diagonal, U_1 and U_2 are upper triangular matrices with 1's on the diagonal, and D_1 and D_2 are diagonal matrices. Premultiplying by L_1^{-1} and postmultiplying by U_2^{-1}, we get the equality $D_1U_1U_2^{-1} = L_1^{-1}L_2D_2$. What can you say about the matrix $L_1^{-1}L_2D_2$? And what about $D_1U_1U_2^{-1}$? Continue the proof from here on.